

COMP/CS 605: Introduction to Parallel Computing

Topic : MPI:: Point to Point and Collective Communications

Mary Thomas

Department of Computer Science
Computational Science Research Center (CSRC)
San Diego State University (SDSU)

Presented: 02/16/17

Updated: 02/16/17

Table of Contents

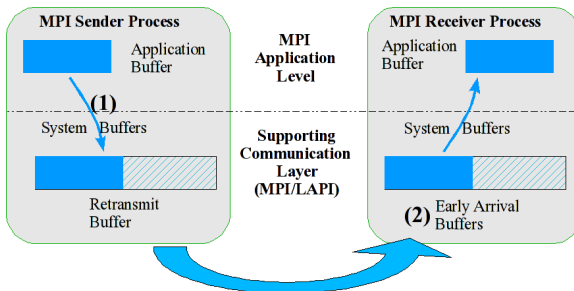
1 MPI Point-to-Point Comms

- MPI: Point-to-Point Communications
- MPI: Send & Receive

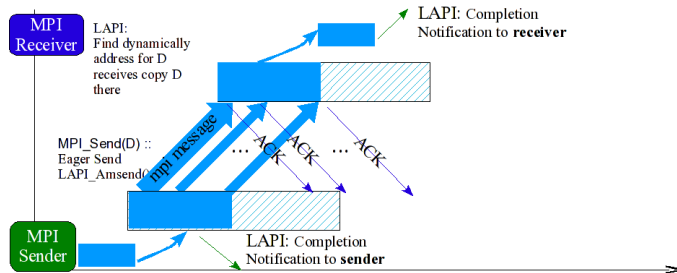
2 MPI Collective Comms

- Parallel Example: Trapezoid Rule
- Tree Structured Communications
- Tree-Structure Global Sums
- Collective Communication: MPI_Reduce
- Collective Communication: MPI_Allreduce
 - Collective Communication: MPI_Broadcast
 - Data Distribution: Vector Add
 - Collective Communication: MPI_Scatter and MPI_Gather

MPI: Point-to-Point Communications



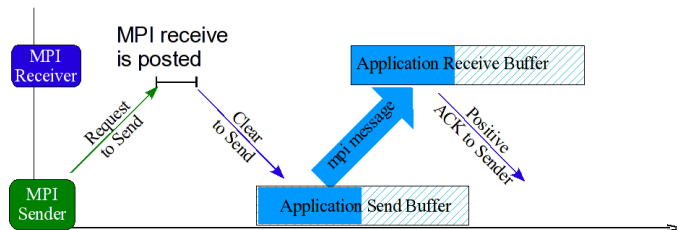
- **Point-to-Point:** uses **MPI_SEND** and **MPI_RECV**.
- Two PEs transfer data from one to the other *only*.
- Blocking: PE #1 posts a **SEND** operation
- Target (PE#2) process posts a **RECEIVE** for data being transferred.



- Smaller messages are sent immediately.
- "Early Arrival" buffer: for messages where RECV has not been posted
- User can [optionally] set env vars:
 - MP_EAGER_LIMIT: maximum message size (default = 32768 bytes)
 - MP_BUFFER_MEM: amount allowed on receiver end.

Image Source: <http://sc.tamu.edu/systems/hydra/hardware.php>

MPI: Message Buffering - Rendezvous Protocol



- For larger messages.
- sender notifies receiver that there is data to be sent.
- Receiver responds when a receive is posted.
- Sender transmits actual message after it receives acknowledgment (ACK).
- Data is saved directly to application receive buffer

MPI: Messages

```
int MPI_Send(void *buf, int count, MPI_Datatype datatype, int  
dest, int tag, MPI_Comm comm)
```

```
int MPI_Recv(void *buf, int count, MPI_Datatype datatype, int  
source, int tag, MPI_Comm comm, MPI_Status *status))
```

- What is the message being sent?
- MPI_Send/Recv message contains the data and specific information about:
 - source task/processor
 - destination task/processor
 - tag for identification of message
 - communicator

Blocking MPI_Send

```
int MPI_Send(void *buf, int count, MPI_Datatype datatype, int
dest, int tag, MPI_Comm comm)
```

```
MPI_Send (
buf          x   Initial address of send buffer (choice)
count        ::  number of elements in send buffer (nonnegative integer)
datatype     ::  datatype of each send buffer element (handle)
dest         ::  rank of destination (integer)
tag          ::  message tag (integer)
comm         ::  communicator (handle)
)
```

Modify mpi_hello.c to do a simple operation (mpi_add.c)

```
if (my_rank != ROOT) {
    /* Create message */
    printf("Greetings from Worker: %d of %d!\n", my_rank, comm_sz);

    /* Send message to ROOT */
    MPI_Send(&my_rank, 1, MPI_INT, ROOT, tag, MPI_COMM_WORLD);
} else {
    sum=0;
    /* Print my message */
    printf("Greetings from Master: %d of %d!\n", my_rank, comm_sz);

    for (int q = 1; q < comm_sz; q++) {

        /* Receive message from process q */
        MPI_Recv(&p, 1, MPI_INT, q, tag, MPI_COMM_WORLD,
                 MPI_STATUS_IGNORE);

        /* Print message from process q */
        printf("P[%d]: RECV Data From Worker node[%d]\n", my_rank, p);
        sum=sum+p;
    }
    printf("P[%d]: SUM of data=%d\n", my_rank, sum);
    /* Shut down MPI */
    MPI_Finalize();
}
```


Output from mpi_add.c

```
[mpi-hello]mthomas% mpicc -o mpi_add mpi_add.c
```

```
[mpi-hello]mthomas% mpirun -np 4 ./mpi_add
```

```
Greetings from Master: 0 of 4!
```

```
Greetings from Worker: 3 of 4!
```

```
P[3]: SUM of data=0
```

```
Data From Proc: 1
```

```
Data From Proc: 2
```

```
Data From Proc: 3
```

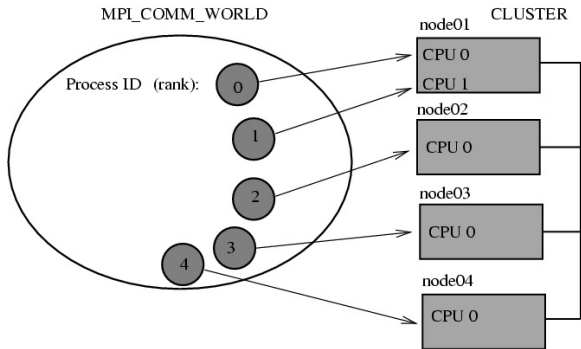
```
Greetings from Worker: 1 of 4!
```

```
P[1]: SUM of data=0
```

```
Greetings from Worker: 2 of 4!
```

```
P[2]: SUM of data=0
```

```
P[0]: SUM of data=6
```



Communication on a multimode cluster with multiple PE's

Program That Might Hang

```
printf("Greetings from process %d of %d!\n", my_rank, comm_sz);
if (my_rank == 0) {
    printf("P[%d] Ready to RECV Data From %d nodes.\n", my_rank, comm_sz)
    for (int q = 1; q < comm_sz; q++) {
        MPI_Recv(&p, 1, MPI_INT, q, tag, MPI_COMM_WORLD,
                 MPI_STATUS_IGNORE);
        printf("Data From Proc: %d\n", p);
        sum=sum+p;
        printf("P[%d] Ready to SEND Data to Node: %d.\n", my_rank, q);
        MPI_Send(&q, 1, MPI_INT, q, tag, MPI_COMM_WORLD);
    }
} else {
    printf("P[%d] Ready to RECV data from Proc %d.\n", my_rank, ROOT);
    MPI_Recv(&p, 1, MPI_INT, ROOT, tag, MPI_COMM_WORLD,
             MPI_STATUS_IGNORE);
    /* Create message */
    printf("P[%d] Ready to SEND my data to: %d.\n", my_rank, ROOT);
    MPI_Send(&my_rank, 1, MPI_INT, ROOT, tag, MPI_COMM_WORLD);
}
printf("P[%d]: SUM of data=%d\n", my_rank, sum);
/* Shut down MPI */
MPI_Finalize();
```

Output from mpi_add_hang.c: NP=4 runs fine

```
[mpi-hello] mthomas% mpicc -o mpi_add_hang mpi_add_hang.c
[mpi-hello] mthomas% mpirun -np 4 ./mpi_add_hang
[mpi-hello] mthomas% mpicc -o mpi_add_hang mpi_add_hang.c
[mpi-hello] mthomas% mpirun -np 4 ./mpi_add_hang
Greetings from process 0 of 4!
P[0] Ready to RECV Data From 4 nodes.
Greetings from process 1 of 4!
P[1] Ready to RECV data from Proc 0.
Greetings from process 2 of 4!
P[2] Ready to RECV data from Proc 0.
Greetings from process 3 of 4!
P[3] Ready to RECV data from Proc 0.
^C Ctrl-C caught... cleaning up processes
```

Output from mpi_add_hang.c: NP=16 hangs

```
[gidget mthomas]% mpirun -np 16 ./mpi_add_hang
Greetings from process 1 of 16!
P[1] Ready to RECV data from Proc 0.
Greetings from process 4 of 16!
P[4] Ready to RECV data from Proc 0.
Greetings from process 6 of 16!
P[6] Ready to RECV data from Proc 0.
Greetings from process 8 of 16!
P[8] Ready to RECV data from Proc 0.
Greetings from process 10 of 16!
P[10] Ready to RECV data from Proc 0.
Greetings from process 12 of 16!
P[12] Ready to RECV data from Proc 0.
Greetings from process 14 of 16!
P[14] Ready to RECV data from Proc 0.
Greetings from process 15 of 16!
P[15] Ready to RECV data from Proc 0.
Greetings from process 2 of 16!
P[2] Ready to RECV data from Proc 0.
Greetings from process 9 of 16!
P[9] Ready to RECV data from Proc 0.
Greetings from process 11 of 16!
P[11] Ready to RECV data from Proc 0.
Greetings from process 3 of 16!
P[3] Ready to RECV data from Proc 0.
Greetings from process 7 of 16!
P[7] Ready to RECV data from Proc 0.
Greetings from process 13 of 16!
P[13] Ready to RECV data from Proc 0.
Greetings from process 0 of 16!
P[0] Ready to RECV Data From 16 nodes.
Greetings from process 5 of 16!
P[5] Ready to RECV data from Proc 0.
```

MPI: Point-to-Point Communications

```

Terminal — top — 127x41
Processes: 195 total, 18 running, 2 stuck, 175 sleeping, 1060 threads
Load Avg: 13.60, 5.06, 2.20 CPU usage: 75.74% user, 24.25% sys, 0.0% idle SharedLibs: 17M resident, 5944K data
MemRegions: 43171 total, 3532M resident, 147M private, 2430M shared.
PhysMem: 2283M wired, 6184M active, 1291M inactive, 9758M used, 6617M free.
VM: 423G vsize, 1053M framework vsize, 6652096(0) pageins, 0(0) pageouts. Networks: packets: 871787/798M in, 66
Disks: 674727/8487M read, 545880/11G written.

PID  COMMAND      %CPU  TIME    #TH   #WQ   #POR  #MREG  RPRVT  RSHRD   RSIZE  VPRVT   VSIZE  PGRP  PPID  STATE   UID
6130  Grab          4.9   00:00.49  4     3    161+  149+   5496K+ 46M+   15M+   84M+   2530M+ 6130  154   sleeping 501
6127  mdworker      0.0   00:01.07  2     1     56   87    5992K  18M    11M    66M    2471M 6127  149   sleeping 89
6126  mdworker      0.0   00:01.26  2     1     53   86    5620K  19M    11M    57M    2469M 6126  154   sleeping 501
6125  top           23.0   00:06.08  1/1   0     27   30    1848K  216K   2592K  17M    2376M 6125  6079  running  0
6124  mdworker      0.0   00:00.07  3     1     56   71    1232K  13M    5832K  53M    2424M 6124  154   sleeping 501
6123  pdftex        0.0   00:03.36  1     0     18   57     39M   1656K  40M    117M   2485M 6123  5751  sleeping 501
6122  mpi_add_hang  51.8   00:45.83  1/1   0     18   36     324K  31M    1564K  18M    2457M 6105  6106  running  501
6121  mpi_add_hang  47.9   00:46.31  1/1   0     18   36     324K  31M    1564K  18M    2457M 6105  6106  running  501
6120  mpi_add_hang  49.4   00:45.94  1/1   0     18   36     324K  31M    1564K  18M    2457M 6105  6106  running  501
6119  mpi_add_hang  47.6   00:46.07  1/1   0     18   36     324K  31M    1564K  18M    2457M 6105  6106  running  501
6118  mpi_add_hang  45.8   00:45.78  1/1   0     18   36     324K  31M    1564K  18M    2457M 6105  6106  running  501
6117  mpi_add_hang  44.9   00:45.70  1/1   0     18   36     324K  31M    1564K  18M    2457M 6105  6106  running  501
6116  mpi_add_hang  46.3   00:46.18  1/1   0     18   36     324K  31M    1564K  18M    2457M 6105  6106  running  501
6115  mpi_add_hang  47.0   00:46.08  1/1   0     18   36     324K  31M    1564K  18M    2457M 6105  6106  running  501
6114  mpi_add_hang  51.2   00:46.07  1/1   0     18   36     324K  31M    1568K  18M    2457M 6105  6106  running  501
6113  mpi_add_hang  47.4   00:46.10  1/1   0     18   36     324K  31M    1568K  18M    2457M 6105  6106  running  501
6112  mpi_add_hang  46.7   00:46.11  1/1   0     18   36     324K  31M    1568K  18M    2457M 6105  6106  running  501
6111  mpi_add_hang  50.7   00:45.56  1/1   0     18   36     324K  31M    1568K  18M    2457M 6105  6106  running  501
6110  mpi_add_hang  49.4   00:46.15  1/1   0     18   36     324K  31M    1568K  18M    2457M 6105  6106  running  501
6109  mpi_add_hang  46.7   00:46.26  1/1   0     18   36     324K  31M    1568K  18M    2457M 6105  6106  running  501
6108  mpi_add_hang  44.7   00:46.10  1/1   0     18   36     324K  31M    1564K  18M    2457M 6105  6106  running  501
6107  mpi_add_hang  46.0   00:46.07  1/1   0     18   36     324K  31M    1564K  18M    2457M 6105  6106  running  501
6106  hydra_pmi_pr  0.0   00:00.00  1     0     17   27    204K   224K   712K   9768K  2376M 6105  6105  sleeping 501
6105  mpiexec.hydr  0.0   00:00.00  1     0     18   28    232K   228K   828K   9644K  2376M 6105  6084  sleeping 501

```

Blocking MPI_SendRecv

```
int MPI_Sendrecv( void *sendbuf, int sendcount, MPI_Datatype sendtype,  
                  int dest, int sendtag,  
                  void *recvbuf, int recvcount, MPI_Datatype recvtype,  
                  int source, int recvtag,  
                  MPI_Comm comm, MPI_Status *status)
```

```
MPI_SendRecv (  
sendbuf           :: Initial address of send buffer (choice)  
sendcount         :: number of elements in send buffer (nonnegative integer)  
sendtype          :: datatype of each send buffer element (handle)  
dest              :: rank of destination (integer)  
sendtag           :: message tag (integer)  
recvbuf           :: Initial address of send buffer (choice)  
recvcount         :: number of elements in send buffer (nonnegative integer)  
recvtype          :: datatype of each send buffer element (handle)  
source            :: rank of destination (integer)  
recvtag           :: message tag (integer)  
comm              :: communicator (handle)  
)
```

Code: mpi_add_sendrecv.c

```
!--Exchange messages
    tag1=1; tag2=2
    printf("Greetings from process %d of %d!\n", my_rank, comm_sz);
    if (my_rank == 0) {
        sum=0;
        for (int q = 1; q < comm_sz; q++) {
            printf("P[%d] Master node ready for SendRecv Op with
                    Worker[%d] \n", my_rank, q);

            sbuf=q*q;
            MPI_Sendrecv(&sbuf, 1, MPI_INT, q, tag1,
                        &p, 1, MPI_INT, q, tag2, MPI_COMM_WORLD,
                        &status);

            sum=sum+p;
        }
    } else {
        printf("P[%d] Worker node ready for SendRecv Op with Master[%d].\n",
                my_rank, ROOT);
        MPI_Sendrecv(&my_rank, 1, MPI_INT, ROOT, tag2,
                    &rbuf, 1, MPI_INT, ROOT, tag1, MPI_COMM_WORLD,
                    &status);
    }
    printf("P[%d]: My rbuf=[%d], my sum=[%d]\n", my_rank, rbuf, sum);
    MPI_Finalize(); /* Shut down MPI */
```


MPI Point-to-Point Comms

MPI: Send & Receive

```
/* listing: mpi.add.c */
#include <stdio.h>
#include <string.h> /* For strlen */
#include <mpi.h> /* For MPI functions, etc */
const int MAX_STRING = 100;
int main(void) {
    char greeting[MAX_STRING]; /* String storing message */
    int comm_sz; /* Number of processes */
    int my_rank; /* My process rank */
    int p, sum, tag=1, ROOT=0;
    MPI_Init(NULL, NULL); /* Start up MPI */
    MPI_Comm_size(MPI_COMM_WORLD, &comm_sz); /* Get the number of processes */
    /* Get my rank among all the processes */
    MPI_Comm_rank(MPI_COMM_WORLD, &my_rank);
    if (my_rank != ROOT) {
        printf("Greetings from Worker: %d of %d!\n", my_rank, comm_sz);
        /* Send message to ROOT */
        MPI_Send(&my_rank, 1, MPI_INT, ROOT, tag, MPI_COMM_WORLD);
    } else {
        /* Print my message */
        printf("Greetings from Master: %d of %d!\n", my_rank, comm_sz);
        for (int q = 1; q < comm_sz; q++) {
            /* Receive message from process q */
            MPI_Recv(&p, 1, MPI_INT, q, tag, MPI_COMM_WORLD, MPI_STATUS_IGNORE);
            printf("P[%d]: RECV Data From Worker node[%d]\n", my_rank, p);
            sum=sum+p;
        }
    }
    printf("P[%d]: SUM of data=%d\n", my_rank, sum);
    /* Shut down MPI */
    MPI_Finalize();
}
```

```
/* FULL CODE LISTING (slide 1/2)
 * File:
 *   mpi.add.sendrecv.c
 * modified by MThomas based on mpi_hello.c
 * Purpose:
 *   A "add,world" program that uses MPI
 *
 * Compile:
 *   mpicc -g -Wall -std=C99 -o mpi_add mpi_add.c
 */

#include <stdio.h>
#include <string.h> /* For strlen */
#include <mpi.h>    /* For MPI functions, etc */
const int MAX_STRING = 100;
int main(void) {
    char    greeting[MAX_STRING]; /* String storing message */
    int     comm_sz;              /* Number of processes */
    int     my_rank;              /* My process rank */
    int     p,q, rbuf,sbuf, sum, ROOT=0;
    int     tag1=1, tag2=2;
    MPI_Status status;

    /* Start up MPI */
    MPI_Init(NULL, NULL);

    /* Get the number of processes */
    MPI_Comm_size(MPI_COMM_WORLD, &comm_sz);

    /* Get my rank among all the processes */
    MPI_Comm_rank(MPI_COMM_WORLD, &my_rank);
```

```
printf("Greetings from process %d of %d!\n", my_rank, comm_sz);
if (my_rank == 0) {
    sum=0;
    for (int q = 1; q < comm_sz; q++) {
        printf("P[%d] Master node ready for SendRecv Op with Worker[%d] \n", my_rank, q);
        sbuf=q*q;
        /*
        */
        MPI_Sendrecv(&sbuf, 1, MPI_INT, q, tag1,
                     &p, 1, MPI_INT, q, tag2, MPI_COMM_WORLD,
                     &status);
        sum=sum+p;
    }
} else {
    printf("P[%d] Worker node ready for SendRecv Op with Master[%d].\n", my_rank, ROOT);
    /*
    */
    MPI_Sendrecv(&my_rank, 1, MPI_INT, ROOT, tag2,
                 &rbuf, 1, MPI_INT, ROOT, tag1, MPI_COMM_WORLD,
                 &status);
}
printf("P[%d]: My rbuf=[%d], my sum=[%d]\n", my_rank, rbuf, sum);
/* Shut down MPI */
MPI_Finalize();

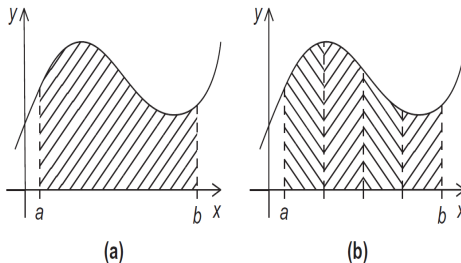
return 0;
} /* main */
```

Output from mpi_add_sendrecv.c

```
[mpi-hello]mthomas% mpicc -o mpi_add_hang mpi_add_hang.c
[mpi-hello]mthomas% mpirun -np 4 ./mpi_add_hang
[mpi-hello]mthomas% mpirun -np 4 ./mpi_add_sendrecv | sort | grep "Master n
P[0] Master node ready for SendRecv Op with Worker[1]
P[0] Master node ready for SendRecv Op with Worker[2]
P[0] Master node ready for SendRecv Op with Worker[3]
[mpi-hello]mthomas% mpirun -np 4 ./mpi_add_sendrecv | sort | grep "Worker n
P[1] Worker node ready for SendRecv Op with Master[0].
P[2] Worker node ready for SendRecv Op with Master[0].
P[3] Worker node ready for SendRecv Op with Master[0].
[mpi-hello]mthomas% mpirun -np 4 ./mpi_add_sendrecv | sort | grep rbuf
P[0]: My rbuf=[0], my sum=[6]
P[1]: My rbuf=[1], my sum=[0]
P[2]: My rbuf=[4], my sum=[0]
P[3]: My rbuf=[9], my sum=[0]
```

Recall: The Trapezoid Rule for Numerical Integration

Solve the Integral: $\int_a^b F(x)dx$

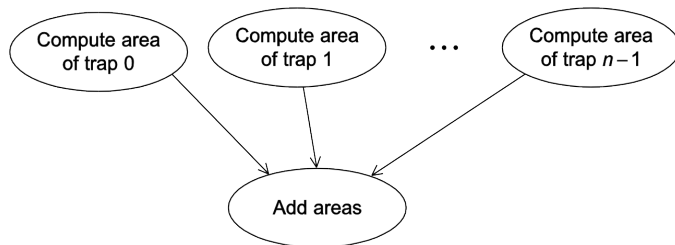


Where $F(x)$ can be any function of x : $f(x^2)$, $f(x^3)$ See Pacheco (2011), Ch3.

Parallelizing the Trapezoidal Rule

- Partition problem solution into tasks.
- Identify communication channels between tasks.
- Aggregate tasks into composite tasks.
- Map composite tasks to cores

Tasks and communications for Trapezoidal Rule



**Master Node collects sums using `MPI_SEND` / `MPI_RECV`
 $\vartheta(np - 1)$, where np is the number of PEs**

Output shows 7 Send/Recv pairs: $\vartheta(np - 1) = \vartheta(7)$, for $np = 8$

```
[mthomas]% mpirun -np 8 ./mpi_trap2
```

```
Enter a, b, and n
```

```
1 50 100
```

```
PE[0] RECV from PE[1]: local estimate=584.199266
```

```
PE[0] RECV from PE[2]: local estimate=1466.537954
```

```
PE[0] RECV from PE[3]: local estimate=2755.471586
```

```
PE[0] RECV from PE[4]: local estimate=4451.000162
```

```
PE[0] RECV from PE[5]: local estimate=6553.123682
```

```
PE[0] RECV from PE[6]: local estimate=9061.842146
```

```
PE[0] RECV from PE[7]: local estimate=11977.155554
```

```
PE[1] SEND: For 12 trapezoids, local estimate=584.199266
```

```
PE[2] SEND: For 12 trapezoids, local estimate=1466.537954
```

```
PE[3] SEND: For 12 trapezoids, local estimate=2755.471586
```

```
PE[4] SEND: For 12 trapezoids, local estimate=4451.000162
```

```
PE[5] SEND: For 12 trapezoids, local estimate=6553.123682
```

```
PE[6] SEND: For 12 trapezoids, local estimate=9061.842146
```

```
PE[7] SEND: For 12 trapezoids, local estimate=11977.155554
```

```
With n = 100 trapezoids, our estimate
```

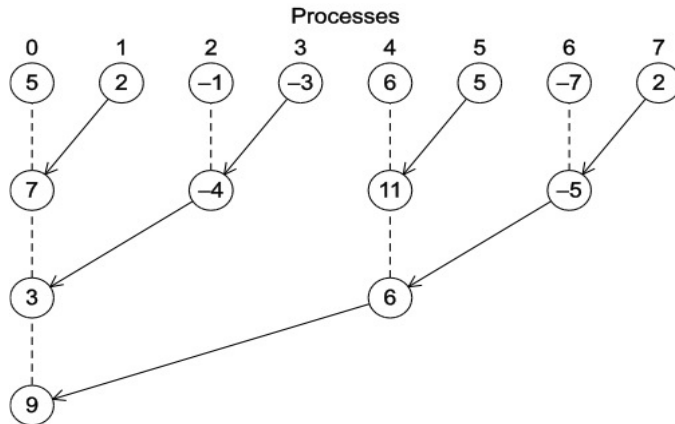
```
of the integral from 1.000000 to 50.000000 = 3.695778587200000e+04
```


Tree-structured communication

Reduces the Number of Communications:

1. In the first phase:
 - (a) Process 1 sends to 0; 3 sends to 2; 5 sends to 4; and 7 sends to 6.
2. Next phase:
 - (a) Processes 0, 2, 4, and 6 add in the received values.
 - (b) Processes 2 and 6 send their new values to processes 0 and 4, respectively.
 - (c) Processes 0 and 4 add the received values into their new values
3. Final phase:
 - (a) Process 4 sends its newest value to process 0.
 - (b) Process 0 adds the received value to its newest value.

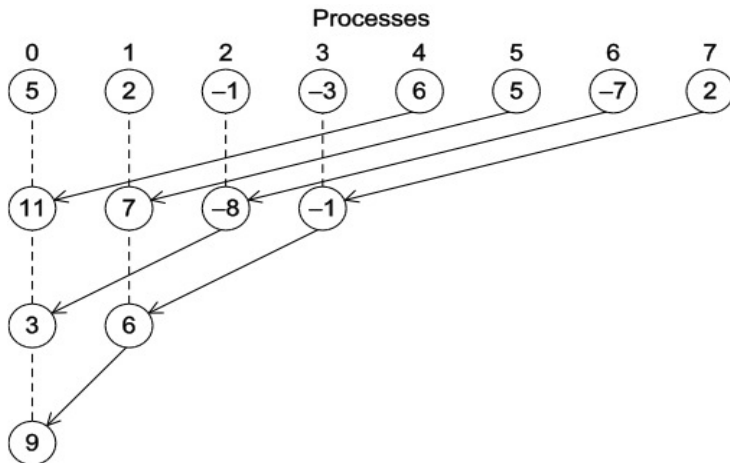
Tree-structured Global Sum V1



Communication pattern used by 1st version of trap.c algorithm:

7 messages, 7 adds by node 0.

With tree-structured Comm: master has 3 messages and 3 adds.



Collective Communication: MPI_Reduce

```
int MPI_Reduce(  
    void*      input_data_p    /* in */,  
    void*      output_data_p   /* out */,  
    int        count           /* in */,  
    MPI_Datatype datatype      /* in */,  
    MPI_Op      operator        /* in */,  
    int        dest_process     /* in */,  
    MPI_Comm    comm           /* in */);
```

```
MPI_Reduce(&local_int, &total_int, 1, MPI_DOUBLE, MPI_SUM, 0,  
          MPI_COMM_WORLD);
```

```
double local_x[N], sum[N];
```

```
...
```

```
MPI_Reduce(local_x, sum, N, MPI_DOUBLE, MPI_SUM, 0,  
          MPI_COMM_WORLD);
```

Operator passed as an argument. Count > 1 supports arrays

Predefined reduction operators in MPI

Operation Value	Meaning
MPI_MAX	Maximum
MPI_MIN	Minimum
MPI_SUM	Sum
MPI_PROD	Product
MPI_LAND	Logical and
MPI_BAND	Bitwise and
MPI_LOR	Logical or
MPI_BOR	Bitwise or
MPI_LXOR	Logical exclusive or
MPI_BXOR	Bitwise exclusive or
MPI_MAXLOC	Maximum and location of maximum
MPI_MINLOC	Minimum and location of minimum

Collective vs. Point-to-Point Communications

- All communicator processes must call same collective function.
 - e.g.: program attempts to match a call to `MPI.Reduce` on `PE(i)` with a call to `MPI.Recv` on `PE(j)` will cause program will hang or crash.
- Arguments passed by each process to an MPI collective communication must be compatible.
 - e.g.: `PE(i)` passes in 0 as the `dest_process` and another passes in 1, then the outcome of a call to `MPI.Reduce` causes code to hang or crash.
- The `output_data_p` argument is only used on `dest_process`.
- All processes need to pass argument corresponding to `output_data_p`
- Point-to-point communications are matched on the basis of tags and communicators.
- Collective communications matched by communicator and order called

Example: What happens when we have multiple calls to MPI_Reduce?

Time	Process 0	Process 1	Process 2
0	a = 1; c = 2	a = 1; c = 2	a = 1; c = 2
1	MPI_Reduce(&a, &b, ...)	MPI_Reduce(&c, &d, ...)	MPI_Reduce(&a, &b, ...)
2	MPI_Reduce(&c, &d, ...)	MPI_Reduce(&a, &b, ...)	MPI_Reduce(&c, &d, ...)

tinit : $p0[a = 0, b = 0, c = 0, d = 0]; p1[a = 0, b = 0, c = 0, d = 0]; p2[a = 0, b = 0, c = 0, d = 0]$
t0 : $p0[a = 1, b = 0, c = 2, d = 0]; p1[a = 1, b = 0, c = 2, d = 0]; p2[a = 1, b = 0, c = 2, d = 0]$
t1 : $p0[a = 1, b = 1, c = 2, d = 0]; p1[a = 1, b = 0, c = 2, d = 2]; p2[a = 1, b = 1, c = 2, d = 0]$
on p0 : $b = P0(a) + P1(c) + P2(a) = 1 + 2 + 1 = 4, d = 0$
t2 : $p0[a = 1, b = 2, c = 2, d = 2]; p1[a = 1, b = 1, c = 2, d = 0]; p2[a = 1, b = 0, c = 2, d = 2]$
on p0 : $d = P0(c) + P1(a) + P2(c) = c + a + c = 2 + 1 + 2 = 5$

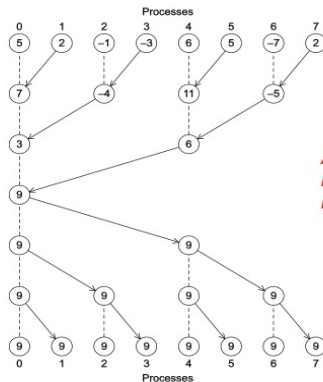
Collective vs. Point-to-Point Communications

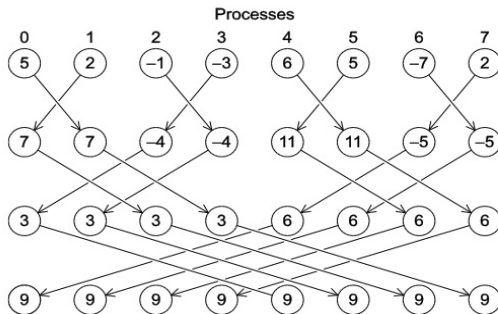
- Suppose that each process calls `MPI.Reduce` with operator `MPI.SUM`, and destination process 0.
- At first glance, it might seem that after the two calls to `MPI.Reduce`, the value of b will be 3, and the value of d will be 6.
- However, the names of the memory locations are irrelevant to the matching of the calls to `MPI.Reduce`.
- The order of the calls will determine the matching so the value stored in b will be $1+2+1 = 4$, and the value stored in d will be $2+1+2 = 5$.

MPI_Allreduce

- Useful in a situation in which all of the processes need the result of a global sum in order to complete some larger computation.

```
int MPI_Allreduce(  
    void*      input_data_p    /* in */,  
    void*      output_data_p   /* out */,  
    int        count           /* in */,  
    MPI_Datatype datatype      /* in */,  
    MPI_Op      operator       /* in */,  
    MPI_Comm    comm           /* in */);
```



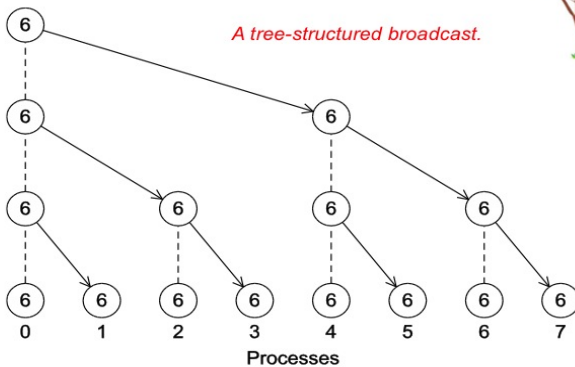


A butterfly-structured global sum.

Broadcast

- Data belonging to a single process is sent to all of the processes in the communicator.

```
int MPI_Bcast(  
    void*      data_p      /* in/out */ ,  
    int        count       /* in      */ ,  
    MPI_Datatype datatype   /* in      */ ,  
    int        source_proc  /* in      */ ,  
    MPI_Comm    comm       /* in      */ );
```



A version of Get_input that uses MPI_Bcast

```
void Get_input(  
    int      my_rank    /* in  */,  
    int      comm_sz    /* in  */,  
    double*  a_p        /* out */,  
    double*  b_p        /* out */,  
    int*     n_p        /* out */) {  
  
    if (my_rank == 0) {  
        printf("Enter a, b, and n\n");  
        scanf("%lf %lf %d", a_p, b_p, n_p);  
    }  
    MPI_Bcast(a_p, 1, MPI_DOUBLE, 0, MPI_COMM_WORLD);  
    MPI_Bcast(b_p, 1, MPI_DOUBLE, 0, MPI_COMM_WORLD);  
    MPI_Bcast(n_p, 1, MPI_INT, 0, MPI_COMM_WORLD);  
} /* Get_input */
```

Data distributions

$$\begin{aligned}\mathbf{x} + \mathbf{y} &= (x_0, x_1, \dots, x_{n-1}) + (y_0, y_1, \dots, y_{n-1}) \\ &= (x_0 + y_0, x_1 + y_1, \dots, x_{n-1} + y_{n-1}) \\ &= (z_0, z_1, \dots, z_{n-1}) \\ &= \mathbf{z}\end{aligned}$$

Compute a vector sum.

Serial implementation of vector addition

```
void Vector_sum(double x[], double y[], double z[], int n) {  
    int i;  
  
    for (i = 0; i < n; i++)  
        z[i] = x[i] + y[i];  
} /* Vector_sum */
```

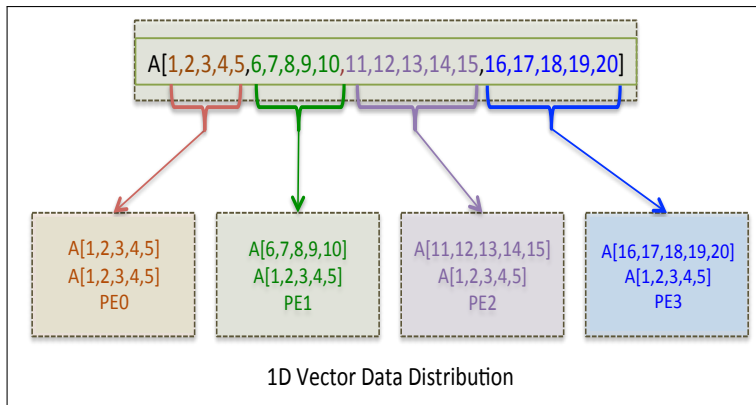

Different partitions of a 12-component vector among 3 processes

Process	Components											
	Block				Cyclic				Block-cyclic Blocksize = 2			
0	0	1	2	3	0	3	6	9	0	1	6	7
1	4	5	6	7	1	4	7	10	2	3	8	9
2	8	9	10	11	2	5	8	11	4	5	10	11

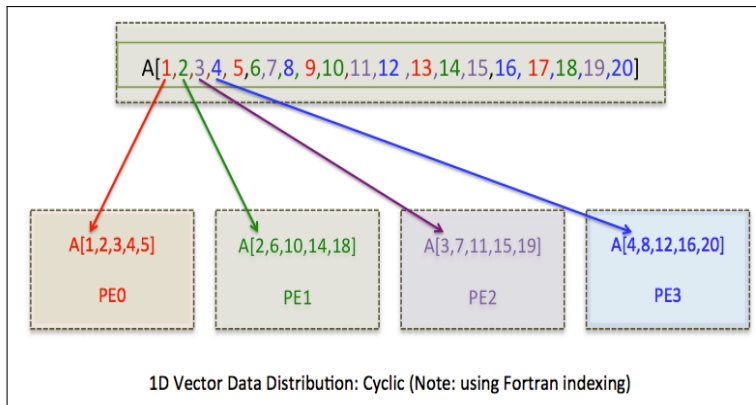
Partitioning options

- Block partitioning
 - Assign blocks of consecutive components to each process.
- Cyclic partitioning
 - Assign components in a round robin fashion.
- Block-cyclic partitioning
 - Use a cyclic distribution of blocks of components.

Block Partitioning of a 20 element vector across 4 processors.



Cyclic Partitioning of a 20 element vector across 4 processors.



Parallel implementation of vector addition

```
void Parallel_vector_sum(  
    double local_x[] /* in */,  
    double local_y[] /* in */,  
    double local_z[] /* out */,  
    int local_n /* in */) {  
    int local_i;  
  
    for (local_i = 0; local_i < local_n; local_i++)  
        local_z[local_i] = local_x[local_i] + local_y[local_i];  
} /* Parallel_vector_sum */
```

Scatter

- MPI_Scatter can be used in a function that reads in an entire vector on process 0 but only sends the needed components to each of the other processes.

```
int MPI_Scatter(  
    void*      send_buf_p /* in */,  
    int        send_count /* in */,  
    MPI_Datatype send_type /* in */,  
    void*      recv_buf_p /* out */,  
    int        recv_count /* in */,  
    MPI_Datatype recv_type /* in */,  
    int        src_proc   /* in */,  
    MPI_Comm   comm       /* in */);
```

Note: the calculation of buffers and counts must be done by programmer.

Reading and distributing a vector

```
void Read_vector(  
    double    local_a[]    /* out */,  
    int       local_n      /* in  */,  
    int       n            /* in  */,  
    char      vec_name[]   /* in  */,  
    int       my_rank      /* in  */,  
    MPI_Comm  comm         /* in  */) {  
  
    double* a = NULL;  
    int i;  
  
    if (my_rank == 0) {  
        a = malloc(n*sizeof(double));  
        printf("Enter the vector %s\n", vec_name);  
        for (i = 0; i < n; i++)  
            scanf("%lf", &a[i]);  
        MPI_Scatter(a, local_n, MPI_DOUBLE, local_a, local_n, MPI_DOUBLE,  
                    0, comm);  
        free(a);  
    } else {  
        MPI_Scatter(a, local_n, MPI_DOUBLE, local_a, local_n, MPI_DOUBLE,  
                    0, comm);  
    }  
} /* Read_vector */
```

All nodes call *MPI_Scatter*; only the master node has a value for *a*; the destination processors store the value of *a* into local variable *local_n*

Gather

- Collect all of the components of the vector onto process 0, and then process 0 can process all of the components.

```
int MPI_Gather(  
    void*          send_buf_p    /* in */,  
    int            send_count    /* in */,  
    MPI_Datatype   send_type     /* in */,  
    void*          recv_buf_p    /* out */,  
    int            recv_count    /* in */,  
    MPI_Datatype   recv_type     /* in */,  
    int            dest_proc     /* in */,  
    MPI_Comm       comm         /* in */);
```


Print a distributed vector (1)

```
void Print_vector(  
    double    local_b[]    /* in */,  
    int       local_n      /* in */,  
    int       n            /* in */,  
    char      title[]      /* in */,  
    int       my_rank      /* in */,  
    MPI_Comm  comm        /* in */) {  
  
    double* b = NULL;  
    int i;
```

Print a distributed vector (2)

```
if (my_rank == 0) {
    b = malloc(n*sizeof(double));
    MPI_Gather(local_b, local_n, MPI_DOUBLE, b, local_n, MPI_DOUBLE,
               0, comm);
    printf("%s\n", title);
    for (i = 0; i < n; i++)
        printf("%f ", b[i]);
    printf("\n");
    free(b);
} else {
    MPI_Gather(local_b, local_n, MPI_DOUBLE, b, local_n, MPI_DOUBLE,
               0, comm);
}
} /* Print_vector */
```

Allgather

- Concatenates the contents of each process' `send_buf_p` and stores this in each process' `recv_buf_p`.
- As usual, `recv_count` is the amount of data being received from each process.

```
int MPI_Allgather(  
    void*      send_buf_p /* in */,  
    int        send_count /* in */,  
    MPI_Datatype send_type /* in */,  
    void*      recv_buf_p /* out */,  
    int        recv_count /* in */,  
    MPI_Datatype recv_type /* in */,  
    MPI_Comm   comm       /* in */);
```