STAT 700
Homework 6 Problems
due Wed. Oct. 31

3 Problems. Please follow the Lab report directions off the homework web page and work in HW Groups.

1. (a) Consider the fixed effects two-way ANOVA model:

$$Y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}, \;\; i = 1, 2; j = 1, 2, 3. \tag{1}$$

$\varepsilon_{ij}$ are i.i.d. $N(0, \sigma^2)$ random variables.

Write this model in matrix notation, indicating all individual elements and dimensions of matrices using:

$$\boldsymbol{Y} = \boldsymbol{X\beta} + \boldsymbol{\varepsilon} \tag{2}$$

(b) Now, Consider the linear mixed model (one-way ANOVA with random block effects):

$$Y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}, \;\; i = 1, 2; j = 1, 2, 3. \tag{3}$$

$\varepsilon_{ij}$ are i.i.d. $N(0, \sigma^2)$, $\beta_j$ i.i.d. $N(0, \sigma^2_\beta)$, and $\alpha_i$ are constants.

Write this model in matrix notation, indicating all individual elements and dimensions of matrices using:

$$\boldsymbol{Y} = \boldsymbol{X\beta} + \boldsymbol{Zu} + \boldsymbol{\varepsilon} \tag{4}$$

2. Return to **Dyestuff Data:** (Ref: Davies, 1960) of Homework 5, Problem 3 and the one-way ANOVA model with a single random effect,

$$Y_{ij} = \mu + \alpha_i + \varepsilon_{ij} \tag{5}$$

where $\mu$ is the overall mean level, $\alpha_i$ is the random effect of the ith batch and they are i.i.d. $N(0, \sigma^2_\alpha)$ and $\varepsilon_{ij}$ are i.i.d. $N(0, \sigma^2)$.

(a) We can use the R function `lme` to fit the random-effects model using REML (restricted maximum likelihood, the default). Your call should look like,

```
> fit <- lme(Strength~1, data=dye, random=~1 | Batch)
```

Use the summary function to get the REML estimates for the $\sigma^2$ and $\sigma^2_\alpha$. How do these estimates compare with your estimates from Homework 5, Problem 3 (c)?

(b) The plot function will give you a diagnostic plot for your fit. From this plot, how well does the model fit the data?

(c) Test the hypothesis $\mathrm{H}_0 : \sigma^2_\alpha = 0$ vs $\mathrm{H}_1 : \sigma^2_\alpha \neq 0$, using a LRT. What do you conclude?

3. The concentrations (in nanograms per milliliter) of plasma epinephrine were measured for ten dogs under: (1) isofluorane, (2) halothane, and (3) cyclopropane anesthesia. (Ref: Perry et al, 1974).

We will study **blocking** and we will use data available off the class web page:

`https://edoras.sdsu.edu/~babailey/stat700/dog.dat`

You can use the header information already in the file. Consider the 10 dogs as blocks and the different anesthesia as treatments.

(a) Plot the data using strip charts. Describe any differences that you see.

(b) We will consider the blocks as random effects, so the linear mixed model is:

$$Y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij},\ \ i = 1, \ldots, I; j = 1, \ldots J. \tag{6}$$

where $\alpha_i$ are independent $N(0, \sigma_\alpha^2)$ random variables, $\varepsilon_{ij}$ are independent $N(0, \sigma^2)$ random variables, and $\beta_j$ are constants (subject to $\sum_{j=1}^{J} \beta_j = 0$). The $\alpha_i$ and $\varepsilon_{ij}$ are independent. (Note: This is a different model than in (3).)

Fit a linear mixed effects model with `lme`. Give summary and diagnostics plots of the residuals. What do you conclude? Make sure that a factor is a factor! You can use the as.factor function inside of lme.

(c) Test the hypothesis $\text{H}_0 : \sigma_\alpha^2 = 0$ vs $\text{H}_1 : \sigma_\alpha^2 \neq 0$, using a LRT. (Use the `gls` function). What do you conclude?

(d) The default method with `lme` is REML. Repeat (c) using ML for BOTH fits. How do the $p$-values of the LRT using REML and ML compare?