STAT 496
Homework 4 Problems
due Wed. Oct. 17

5 Problems. Show all work.
Please follow the Lab report directions off the homework web page for R Problems. All problems should be done with RStudio/Sweave. Please work in HW Groups!

1. p. 81: 4.5 (a) and (b)
You have already calculated and the ACF for $k = 0, 1, 2, 3, 4, 5$ lags. Now, calculate the PACF ϕ_{kk} by hand for $k = 0, 1, 2, 4, 4, 5$ lags. Plot the PACF by using the R function ARMAacf with the pacf=TRUE option. See Homework 3 Solutions. (Note: By convention $\phi_{00} = 1$ and is NOT plotted.)

Please see the LaTeX code linked off the HW page that you can insert your final calculations into your .Rnw document. Below is what the LaTeX should look like and you can fill in the ?. This one ? for each model.

(a) AR(1) PACF:
$$\phi_{kk} = \begin{cases} 1 & k = 0 \\ \phi_1 = ? & k = 1 \\ 0 & k \geq 2 \end{cases}$$

(b) AR(1) PACF:
$$\phi_{kk} = \begin{cases} 1 & k = 0 \\ \phi_1 = ? & k = 1 \\ 0 & k \geq 2 \end{cases}$$

2. Similar to Problem 1, but use the following AR(2) model:

$$Y_t = 1.4Y_{t-1} - 0.48Y_{t-2} + e_t$$

where $\{e_t\}$ is a mean zero white noise process with constant variance σ_e^2.

You have already calculated and the ACF for $k = 0, 1, 2, 3, 4, 5$ lags for your Exam. Now, you calculate the PACF ϕ_{kk} by hand for $k = 0, 1, 2, 3, 4, 5$ lags. Plot the PACF by using the R function ARMAacf with the pacf=TRUE option. See Homework 3 Solutions. (Note: By convention $\phi_{00} = 1$ and is NOT plotted.)

Please see the LaTeX code linked off the HW page so that you can insert your final calculations into your .Rnw document. Below is what the LaTeX should look like and you can fill in the two ?.

AR(2) PACF:
$$\phi_{kk} = \begin{cases} 1 & k = 0 \\ \phi_1/(1 - \phi_2) = ? & k = 1 \\ \phi_2 = ? & k = 2 \\ 0 & k \geq 3 \end{cases}$$

3. Use the models in Problem 1 and use the R function `arima` to simulate a time series of length $n = 500$. Use the R `set.seed` function, so that you can reproduce your results. Make a time series plot of the data, sample ACF plot, AND sample PACF. (See the Lab.) You should have 2 simulated datasets. How do the sample PACF functions compare to the ones you calculated in Problem 1? You do not need to make a table. Just look at the sample PACF and indicate if the estimates are "close" to the theoretical values obtained in Problem 1. (Note: By convention $\phi_{00} = 1$ and is not included on the plot)

4. Use the model in Problem 2 and use the R function `arima` to simulate a time series of length $n = 500$. Use the R `set.seed` function, so that you can reproduce your results. Make a time series plot of the data, sample autocorrelation function plot, AND sample PACF. (See the Lab.) You should have one simulated dataset. How does the sample PACF function compare to the one you calculated in Problem 2? You do not need to make a table. Just look at the sample PACF and indicate if the estimates are "close" to the theoretical values obtained in Problem 1. (Note: By convention $\phi_{00} = 1$ and is not included on the plot)

5. p. 146: 6.34