Chapter 2 Random Variables

2.1 Discrete Random Variables

For example: for a flip of a coin $\Omega = \{H, T\}$.

Let X be a function such that $X(\omega) = 0$ if $\omega = T$ and $X(\omega) = 1$ if $\omega = H$.
X is a real-valued function from Ω to $\mathcal{A}$.

Definition. Consider a random experiment with a sample space Ω. A function X that assigns to each $\omega \in \Omega$ one and only one real number $X(\omega) = x$ is called a **random variable**. The space of X is the set of real numbers $\mathcal{A} = \{x : x = X(\omega), \omega \in \Omega\}$.

Example: All possible sequences of coin flips.

Let X be a random variable with space $\mathcal{A}$. Suppose that $\mathcal{A}$ is countable. Let a function $p(x)$ be such that $p(x) > 0$ for $x \in \mathcal{A}$ and

$$\sum_{x \in \mathcal{A}} p(x) = 1.$$

Also, for any event $A \subset \mathcal{A}$ suppose that

$$P(X) = P(X \in A) = \sum_{x \in A} p(x).$$

Then X is called a **random variable of the discrete type** and $p(x)$ is called the **probability mass function or frequency** of X.

Define the function $F(x)$ by $F(x) = P(X \leq x)$ for all x. F is called the **cumulative distribution function (cdf)** of the random variable X.

Properties of F:

1. $0 \leq F(x) \leq 1$
2. $F(x)$ is nondecreasing as x increases
3. $\lim_{x \to -\infty} F(x) = 0$ and $\lim_{x \to \infty} F(x) = 1$
4. $F(x)$ is continuous from the right, i.e., right-continuous

Common Discrete Distributions:

- Bernoulli Random Variables
- The Binomial Distribution
- The Geometric and Negative Binomial Distribution
- The Hypergeometric Distribution
- The Poisson Distribution

2.2 Continuous Random Variables

Let X denote a random variable with a one-dimensional space $\mathcal{A}$, which consists of an interval or a union of intervals. Let $f(x)$ be a nonnegative function that integrates to 1 over the space $\mathcal{A}$ and for any $A \subset \mathcal{A}$

$$P(A) = P(X \in A) = \int_A f(x).$$

Then X is said to be a **random variable of the continuous type** and $f(x)$ is called the **probability density function** of X.

Example: pdf

For continuous random variables, the distribution function is defined by

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(u)du$$

From the Fundamental Theorem of Calculus, if f is continuous at x, $f(x) = F'(x)$.

Also, the cdf can be used to evaluate:

$$P(a \leq X \leq b) = F(b) - F(a)$$

Common Continuous Distributions:

- Uniform Density
- Exponential Density
- Gamma Density
- Normal Distribution